An MIP Approach for scheduling jobs with degradation rate

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Introduction

- ✓ We study the processing time planning of jobs of a system, in which the process time of each job will deteriorate with its starting time
- ✓ Minimizing the total processing completion time.
- ✓ This research develops one application of this model of job scheduling, in which potholes repair time is scheduled
- ✓ we propose an MIP model for repair time planning of potholes of a road network.

Motivation

- ✓ A pothole is a structural failure in an asphalt pavement, caused by the presence of water in the underlying soil structure and the presence of traffic passing over the affected area.
- ✓ Potholes are created when moisture seeps into the pavement, freezes, expands and then thaws

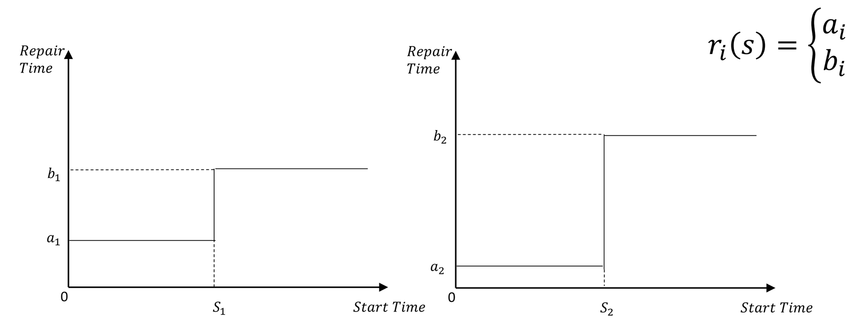
- ✓ Pothole damage has cost U.S. drivers $15 billion in vehicle repairs over the last five years or about $3 billion annually.
- ✓ In the last five years, 16 million drivers across the country have suffered pothole damage to their vehicles
- ✓ On average, American drivers report paying $300 to repair pothole-related vehicle damage

Problem Statement

The decision to patch potholes is influenced by many factors:

- ✓ The level of traffic.
- ✓ The time to begin repair potholes.

$$r_i(s) = \begin{cases} a_i & s \le s_i \\ b_i & s > s_i \end{cases}$$



Problem Statement

- ✓ Repair time is a function of start time
- ✓ Degradation over time
- ✓ Degradation rate = F (flow traffic , time)
- ✓ $\alpha \sim AADT$
- ✓ Repair-duration = $a + \alpha(w)$
- ✓ **Which potholes in which days?**
- ✓ x_{ij}^{t}: a binary variable which is 1 if segment *j* will be repaired after segment *i* during day *t*
- ✓ **When is start time of repair on a given day?**
- ✓ w_i^t: a continuous variable that specifies the start time of repairing segment *i* during time window *t*

Modeling

- ✓ A mixed integer programing model is developed to minimize the completion time of the all potholes

$$min \sum_{t=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n}(a_i + \alpha_i(w_{it} + 24(t-1))x_{ijt} + \sum_{t=1}^{m}\sum_{j=1}^{n}\sum_{i=1}^{n} T_{ij}x_{ijt}$$

Summation over all nodes and time periods

Repair time for node i when are going from i to j

Travel time

MIP Model

$$min \sum_{t=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{n}(a_i + \alpha_i(w_{it} + 24(t-1))x_{ij\mathrm{t}} + \sum_{t=1}^{m}\sum_{j=1}^{n}\sum_{i=1}^{n} T_{ij}x_{ijt}$$

$$\sum_{t=1}^{m}\sum_{j=1}^{n} x_{ijt} = 1 \qquad \forall i$$

$$\sum_{j=1}^{n} x_{0jt} \le 1 \qquad \forall t$$

$$\sum_{i=1}^{n} x_{i(\mathrm{n}+1)t} \le 1 \qquad \forall t$$

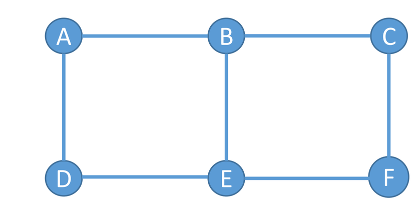
$$\sum_{i=1}^{n} x_{iht} - \sum_{j=1}^{n} x_{hjt} = 0 \qquad \forall h, t$$

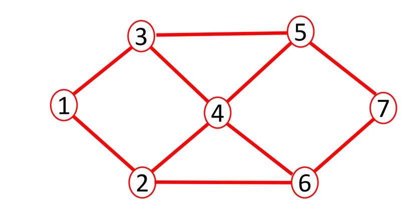
$$w_{it} + (a_i + \alpha_i(w_{it} + 24(t-1)) + T_{ij} \le w_{jt} + M\left(1 - x_{ijt}\right) \quad \forall i, \forall j \;\; i \ne j, \forall t$$

$$w_{0t} = 0 \qquad \forall t$$

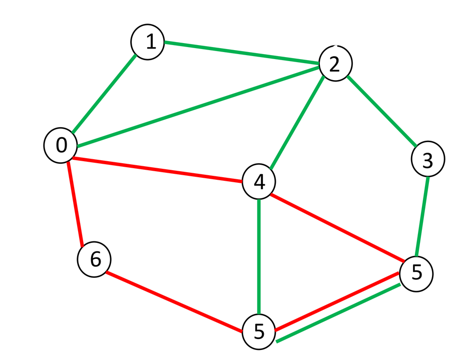
$$0 \le w_{it} \le 8 \qquad \forall i, t$$

Solution Methodology





- ✓ Since we want to visit segments of the roads (links) and our model is proper for visiting nodes, first we have to transfer graph G to a graph H, in which links are the nodes of graph G.

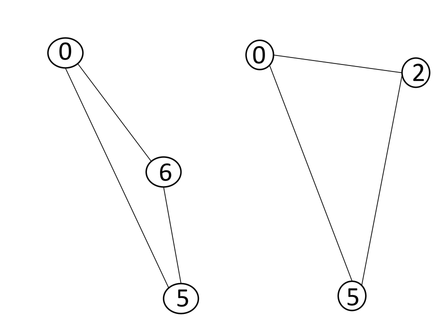


- ✓ Each node is a segment of road that should be visited in one of the time periods.
- ✓ Repair time of each node depends on the start time of repairing.

Constructive Algorithm

Iterative Route Constructive Algorithm

- In each iteration the algorithm build a route to be visited in each day
- First a node with largest degradation rate is added to the route
- In the nest iteration the best node to be added and the best position in the route is determined



$$IU\,(k : i{-}j) = d_{ik} + d_{kj} - d_{ij}$$

$$IU(6{:}0{-}5{-}0) < IU(2{:}0{-}5{-}0)$$

Computational Results

	Premature GUROBI	Time	GUROBI	Time	Greedy	Time	GAP
5	11.57	0.05	11.57	0.08	11.66	0.00	0.99
6	15.63	0.16	15.63	0.22	15.83	0.00	0.99
7	19.31	0.33	19.31	0.38	19.51	0.00	0.99
8	24.08	0.36	24.08	0.76	24.28	0.00	0.99
9	28.85	0.46	28.85	0.93	29.22	0.00	0.99
10	29.63	7.81	29.63	14.79	30.05	0.00	0.99
11	34.24	6.71	34.24	13.00	34.89	0.01	0.98
12	38.13	5.62	38.13	7.59	38.70	0.00	0.99
13	40.11	151.49	40.11	208.55	40.79	0.01	0.98
14	42.07	385.00	42.07	536.65	43.03	0.00	0.98

Chronological Decomposition

Process time	Day 1	Day 2	Day 3	Day 4	Day 5
Job 1	1.4	2.6	3.8	5	6.2
Job 2	1.58	2.06	2.54	3.02	3.5
Job 3	2.12	2.84	3.56	4.28	5
Job 4	0.54	0.78	1.02	1.26	1.5
Job 5	0.91	1.87	2.83	3.79	4.75
Job 6	1.04	1.28	1.52	1.76	2

$$t_{ij} = a_i + \alpha_i(24\,\mathrm{j} + {}^{k}/_{2})$$

a_i (Repair time)	α_i (Degradation rate)
1.2	0.05
1.5	0.02
2	0.03
0.5	0.01
0.75	0.04
1	0.1

$$min \sum_{\mathrm{j}=1}^{m}\sum_{i=1}^{n} \mathrm{t}_{ij}x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad \forall i$$

$$\sum_{j=1}^{n} x_{\mathrm{i}j} \le \frac{m}{n} \qquad \forall t$$

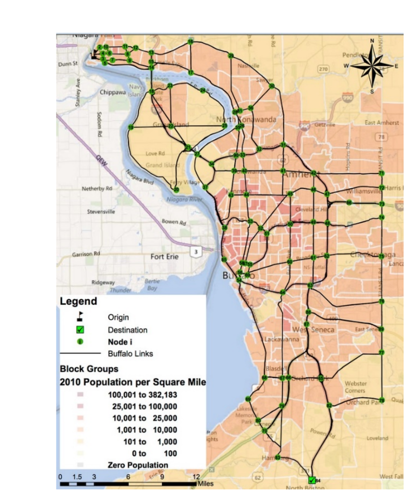
$$x_{ij} \in \{0,1\} \qquad \forall i, \mathrm{j}$$

TSP

Case Study

- ✓ Here a case study of city of buffalo is solved with our proposed model
- ✓ In this graph there are 90 nodes and 148 links
- ✓ The problem is solved for 20 time periods

Day 1	[0, 49, 63, 69, 89, 99, 104, 125, 93, 0]
Day 2	[0, 52, 50, 61, 97, 92, 127, 115, 81, 0]
Day 3	[0, 38, 44, 55, 95, 108, 118, 134, 76, 0]
Day 4	[0, 1, 6, 5, 7, 4, 3, 2, 0]
Day 5	[0, 16, 18, 27, 136, 100, 146, 79, 22, 0]
Day 6	[0, 62, 90, 91, 102, 128, 132, 135, 75, 0]
Day 7	[0, 53, 116, 124, 145, 101, 70, 66, 57, 0]
Day 8	[0, 65, 126, 123, 138, 131, 129, 121, 120, 0]
Day 9	[0, 13, 17, 30, 64, 87, 142, 78, 51, 0]
Day 10	[0, 4, 23, 71, 73, 84, 133, 140, 141, 0]
Day 11	[0, 33, 72, 74, 96, 86, 98, 147, 130, 0]
Day 12	[0, 10, 20, 83, 85, 105, 103, 110, 80, 0]
Day 13	[0, 36, 54, 67, 68, 107, 139, 119, 122, 0]
Day 14	[0, 8, 32, 88, 94, 106, 143, 111, 77, 0]
Day 15	[0, 2, 6, 113, 114, 117, 56, 58, 25, 0]
Day 16	[0, 9, 11, 24, 39, 82, 109, 46, 19, 0]
Day 17	[0, 1, 29, 37, 144, 137, 48, 21, 7, 0]
Day 18	[0, 3, 12, 14, 34, 59, 148, 112, 41, 0]
Day 19	[0, 31, 45, 42, 47, 43, 28, 60, 0]
Day 20	[0, 5, 15, 26, 0]



Conclusion and Future Study

- ✓ We are going to analyze the effect of different parameters on objective function and run time to solve the problem.
- ✓ Future studies can apply the proposed optimization scheme for modeling the similar problems as humanitarian relief from hazard problems , delivery for medication.